Investigation of joint clearance effects on the dynamic performance of a planar 2-DOF pick-and-place parallel manipulator

Xu Li-xin*, Li Yong-gang

School of Mechanical Engineering, Tianjin University of Technology and Education, Tianjin 300222, China

ARTICLE INFO

Abstract

In this study, the effects of joint clearance on the dynamic performance of a planar 2-DOF pick-and-place parallel manipulator are investigated. The parallel manipulator is modeled by multi-body system dynamics. The contact effect in revolute joints with clearance is established by using a continuous analysis approach that is combined with a contact force model considering hysteretic damping. The evaluation of the contact force is based on Hertzian contact theory that accounts for the geometrical and material properties of the contacting bodies. Furthermore, the incorporation of the friction effect in clearance joints is performed using a modified Coulomb friction model. By numerical simulation, variations of the clearance joint’s eccentric trajectory, the joint reaction force, the input torque, the acceleration, and trajectory of the end-effector are used to illustrate the dynamic behavior of the mechanism when multiple clearance revolute joints are considered. The results indicate that the clearance joints present two obvious separation leaps in a complete pick-and-place working cycle of the parallel manipulator, following a collision. The impact induces system vibration and thus reduces the dynamic stability of the system. The joint clearances affect the amplitudes of the joint reaction force, the input torque, and the end-effector’s acceleration, additionally the joint clearances degrade the kinematic and dynamic accuracy of the manipulator’s end-effector. Finally, this study proposes related approaches to decrease the effect of joint clearances on the system’s dynamic properties for such parallel manipulator and prevent “separation-leap-impact” events in clearance joints.

Keywords:
Multi-body dynamics
Parallel manipulator
Clearance joints
Collision
Dynamic accuracy

1. Introduction

The planar 2-DOF pick-and-place parallel mechanism has been widely applied in industrial automation production engineering. It requires high performance with regards running precision, stability, and reliability. With the continuous development of this type of robotic mechanism providing higher speed and precision, the effects of joint clearance on the dynamic performance of the mechanism are subject to increasing research attention. Since the joint clearance changes the force state between mechanism members, the dynamic performance of the mechanism is thereby affected. This effect is more serious especially for high-speed mechanisms. Due to the existence of the joint clearance, a collision will be caused between the hinged components. This collision can cause additional dynamic reactions in mechanism members, which increases the wear of joint elements and decreases the kinematic accuracy of the mechanism, meanwhile, it triggers vibration, noise, etc.

In the past decade, researchers have developed studies of the kinematic accuracy of mechanisms with joint clearance. Zhu and Ting [1] presented a general probability density function based analysis method to examine the performance uncertainty caused by joint clearance in a robot. Based on the general probability density function of the end-point, the distribution functions of the robot end-point for any position with tolerance zone and any joint distribution type can be derived. Ting et al. [2] proposed a novel approach and application of N-bar rotatability laws to determine the uncertain link orientation and point position caused by joint clearance in linkages and manipulators. This method offers a simple and effective model for understanding the effect of each joint clearance on the deviation of the orientation and position of any link in a linkage. Tsai and Lai [3] introduced a method to analyze the transmission performance of linkages with joint clearance. Later, the authors [4] presented an approach for the error analysis of multi-loop mechanisms with joint clearance and six-bar linkages with different specified input links were taken as examples to analyze mechanism positioning error. In these studies, the joint clearance was treated as a virtual link to simplify the model. Parenti-Castelli and Venanzi [5] proposed a technique for studying the kinematic influence of joint clearance on the position and orientation of the links of spatial mechanisms. This method is
suitable for evaluating clearance influence under static and dynamic conditions, when inertial forces or torques acting on each link have to be introduced. Meng et al. [6] developed an error prediction model that is applicable to planar or spatial parallel manipulators that are either over-constrained or non-over-constrained. By using their method, the maximal positional error in a prescribed workspace can be efficiently computed. Erkaya and Uzumay [7] studied joint clearance influence on mechanism path generation and transmission angle. The joint clearance was treated as a massless virtual link and a genetic algorithm approach was used to describe the direction of the joint clearance relative to input link position. Chebbi et al. [8] developed an analytical model capable of quantifying the positional error for a 3-UPU parallel robot as a function of the joint's clearance magnitude, the configuration of the mechanism, and the load acting on the platform. For increasing the positioning accuracy and dynamic performance of a robot mechanism, Bu et al. [9] proposed a method based on trajectory planning to avoid the detachment of joint elements of a manipulator with clearances. Altuzarra et al. [10] introduced a methodology for accuracy analysis of parallel manipulators with joint clearance. The errors in the position of the end effector due to clearance were calculated by means of an approximation to the velocity analysis, while the relative position of the two parts of the joints was determined by means of a dynamic analysis. Frisoli et al. [11] presented a novel method based on screw theory for the analysis of positional accuracy in spatial parallel manipulators with revolute joint clearances. Pandey and Zhang [12] developed an efficient and accurate method for computing system reliability of robotic manipulators with randomness in both joint clearances and dimensions of links. Jawale and Thorat [13] presented an approach for quantifying error due to joint clearance in a 2-DOF mechanism. In this research, the clearance was also modeled as a small link between the nominal links of the mechanism. Chaker et al. [14] analyzed the combined effect of manufacturing errors and clearance on the positional error of the platform of a 3-RCC spherical parallel manipulator. The manufacturing errors and clearance were assumed to be small displacements.

With this body of in-depth research, it is found that the method of assuming joint clearance to behave as a rigid link model cannot effectively describe the contact, separation, and collision states between clearance joint elements, and cannot reveal the impact-contact characteristics in a clearance joint. In recent years, Ravn [15] and Flores et al. [16] proposed a continuous analysis approach to describe clearance revolute joints in rigid multi-body systems based on the contact force model presented by Lankarani and Nikravesh [17]. A planar slider-crank mechanism was used as an example to demonstrate the application of the method. The dynamic behavior of the mechanism and the impact mechanics in a clearance joint were analyzed. Later, Ravn et al. [18], Flores et al. [19–22], Alshaer et al. [23], and Machado et al. [24] introduced the film lubrication effect into the clearance revolute joint model. By simulation and comparison, it is indicated that lubrication can eliminate maxima in the magnitude reaction force of a clearance joint and reduce vibration of the mechanism. In continuation of this research, Flores et al. [25–27], Liu et al. [28], and Megahed and Haroun [29] discussed the dynamic performance of multi-body systems with multiple clearances. It is indicated that the dynamic behavior of a one clearance joint cannot be used as a general case for a mechanical system with multiple clearances. Flores et al. [30], Muvengei et al. [31], and Machado et al. [32] discussed several different compliant contact force models used in the context of multi-body system dynamics to model and analyze contact-impact events. Bai and Zhao [33] studied the dynamic behavior of a slider-crank mechanism with revolute joint clearance: their contact model for revolute joint clearance is established using a new non-linear continuous contact force model, which is a hybrid of the Lankarani-Nikravesh model and an improved elastic foundation model. It is well known that friction in contact influences the dynamic characteristics of clearance joints. In research into friction modeling, Karnopp [34], Haessig and Friedland [35], and Liang [36] proposed several friction models that can be used in the analysis of contact problems. Muvengei et al. [37] analyzed the dynamic response of a slider-crank mechanism when different friction models were adopted on modeling of clearance joints. Flores [38] investigated the wear process in surface interaction of clearance joints under the framework of a multi-body system formulation. The research on modeling of clearance joints is not limited to planar mechanisms with only revolute joints. Reports on the dynamic modeling and simulation of multi-body systems with spatial clearance joints, including spherical and cylindrical joints, are also available in a number of publications [39–40]. Furthermore, Flores et al. [41] presented a method for modeling translational joints with clearance in rigid multi-body systems.

The above literature review shows that dynamic research into multi-body systems with joint clearance is a pressing topic, worthy of further attention. Ravn, Flores, Alshaer, Machado, Liu, Tian, Megahed, Muvengei, and Machado et al. have published more than 20 papers thereon. In this body of the literature, several methods were proposed to establish a model for the clearance joint based on the theoretical dynamic framework for multi-body systems. The efficacy of the proposed clearance joint models was verified by simulating a simple slider-crank, or four-bar, mechanism. Unfortunately, the effect of joint clearance on the dynamic properties and kinematic accuracy of robots in practical engineering applications is rarely studied. In related research, Zhao and Bai [42] analyzed the dynamic characteristics of a space robot manipulator with joint clearance. A non-linear equivalent spring-damper model was established for their contact model of joint clearance and frictional effects were considered using the Coulomb friction model. Shiau et al. [43] presented a non-linear dynamic analysis of a 3-PRS series-parallel mechanism with consideration of joint effects which included flexibility, clearance, and friction. Erkaya [44] investigated the kinematic and dynamic characteristics of a welding robot manipulator with joint clearance. It is verified that the joint clearance can cause degradation of kinematic and dynamic performance of the robot system even if the clearance was small.

Generally, numerical methods are used to solve the equations of motion for constrained multi-body systems. The problem is that numerical solutions are unstable and the original constraint equations are violated by solutions laden with accumulated integration truncation errors. For solving the motion equations efficiently and accurately, Flores et al. [45] analyzed the influence of the variables that affect the violation of constraints including the values of the Baumgarte parameters, the integration method, the time step, and the quality of the initial conditions for the positions. Additionally, the method for the automatic detection of the precise instant of contact in contact-impact analysis, and for adjusting the integration time-step accordingly, was also discussed by the authors [46].

The objective of this work is to investigate the joint clearance effects on the dynamic performance of a planar 2-DOF pick-and-place parallel manipulator. The dynamic model of the manipulator is established based on multi-body system dynamics theory. The variations of the clearance joint’s eccentric trajectory, the joint reaction force, the input torque, the acceleration, and trajectory of the end effector are used to illustrate the dynamic behavior of the mechanism when multiple clearance revolute joints are considered. The remainder of the paper is organized as follows: in Section 2, the approach for modeling a clearance revolute joint in a planar multi-body system is offered, in Section 3, the model of
a typical 2-DOF pick-and-place parallel manipulator with two clearance revolute joints is presented, the dynamic response of the parallel manipulator is analyzed in Section 4, and finally, the conclusions are presented.

2. Modeling of a planar multi-body system with a clearance revolute joint

2.1. Analysis of the methods for modeling revolute joint in a planar multi-body system

Generally speaking, there are two methods used to build the revolute joint connection between rigid bodies in multi-body systems. One is to establish an ideal revolute joint model based on kinematic constraints, namely the effect of joint clearance is ignored, as shown in Fig. 1(a). The other is to build a non-ideal revolute joint model based on force interaction, namely the effect of joint clearance is considered, as shown in Fig. 1(b).

When an ideal revolute joint connection between rigid bodies is established by using kinematic constraints, the constraint equation can be expressed as

\[ \Phi^{(i,j)} = (r_j + A_j s_j^i) - (r_i + A_i s_i^j) = 0 \]

where \( r_i \) and \( r_j \) are the position vectors in the global coordinate system (XOY) that describes the location of the body-fixed coordinates \( (\xi, \eta, \zeta) \) and \( (\xi, \eta, \zeta) \), the vectors \( s_i^j \) and \( s_j^i \) are position vectors in the body-fixed coordinate system that locate the center of the revolute joint, \( A_i \) and \( A_j \) are matrices that transform vectors in the body-fixed coordinates systems into vectors in the global system. It can be seen from Fig. 1(a) that, in the assumption of an ideal revolute joint, the two elements which comprise the revolute joint, are coincident in the center, thus the rigid bodies connected by the joint can only generate relative rotation.

However, there are no ideal joints in practice: manufacturing errors and long-term wear will lead to clearance in joints. The model of a non-ideal revolute joint with clearance can be developed by force interaction method. As shown in Fig. 1(b), when joint elements come into contact, the contact point \( Q \) is stressed by the normal contact force \( f_N \) and the tangential friction \( f_F \) simultaneously. In the global coordinate system, the resultant of the contact force and friction exerted on connected rigid bodies \( i \) and \( j \) can be divided into four component forces: \( f_x^i, f_y^i, f_x^j, \) and \( f_y^j \). Substituting these component forces and their moments on the rigid body center of mass into the multi-body system dynamic equation as generalized external forces, the non-ideal revolute joint connection between rigid bodies can be established.

As shown in Fig. 1(b), the forces \( f_i \) and moment \( T_j \) that act on the center of mass of body \( i \) due to the clearance joint contact can be expressed as follows:

\[ f_i = f_N + f_F = \begin{bmatrix} f_x^i \\ f_y^i \\ f_x^j \\ f_y^j \end{bmatrix}, \quad T_j = (y_j^0 - y_i) f_y^i - (x_j^0 - x_i) f_x^j \]

The corresponding forces and moments applied to the body \( j \) are

\[ f_j = -f_i, \quad T_i = -(y_j^0 - y_i) f_y^i - (x_j^0 - x_i) f_x^j \]

2.2. Modeling a clearance revolute joint in a planar multi-body system

As shown in Fig. 2, when the centers of the joint elements are coincident or the eccentricity \( e \) of the joint elements is smaller than clearance \( c \), the joint elements are separated. When \( e \) is bigger than \( c \), the joint elements come into contact. The key to establishing a clearance revolute joint model lies in how to effectively describe relative movement between the joint elements and make an accurate judgment on the transformation of the contact and separation states in a clearance joint.
and velocities at the contact point have been determined, the normal contact and friction forces can be computed.

An appropriate contact force model is crucial for the precise description of the collision dynamics between bodies. So far, a variety of contact force models have been proposed by: Hertz [47], Zukas et al. [48], Lankarani and Nikravesh [17], Flores et al. [30], Dubowsky and Freudenberg [49], Johnson [50], Radzimovsky [51], and Goldsmith et al. [52]. It is worth noting that only the model developed by Lankarani and Nikravesh [17] is widely used in dynamic studies of mechanical multi-body systems with clearance joints. This is attributable to the simplicity of its contact force model, resulting ease of calculation, applicability to impact in multi-body systems, and rapid convergence due to the inclusion of energy dissipation modelling upon impact. In an academic monograph, Flores [53] compared different contact force models and proved the effectiveness of the model proposed by Lankarani and Nikravesh when describing the dynamic properties in the contact impact of a clearance joint. Besides, he confirmed that Lankarani and Nikravesh's model can be transformed to deal with contact problems between cylindrical surfaces.

Therefore, the continuous contact force model of Lankarani and Nikravesh was used to depict the contact impact between clearance joints in the present study. This model is expressed as

\[ F_N = K\delta^{1.5} \left( 1 + \frac{3(1 - \nu^2)}{4\delta^{-1}} \right) \]

and

\[ h_k = \frac{1 - \nu^2}{2E_k} \]

where \( \delta \) is the relative normal penetration velocity, \( \nu \) is the initial normal impact velocity, \( \nu \) is the restitution coefficient, and \( K \) is a constant dependent on the material properties of the components and their geometry. The constant \( K \) is expressed as

\[ K = \frac{4\pi(h_1 + h_2)}{(R_i + R_j)^{1/2}} \]

and velocities at the contact point have been determined, the normal contact and friction forces can be computed.

An appropriate contact force model is crucial for the precise description of the collision dynamics between bodies. So far, a variety of contact force models have been proposed by: Hertz [47], Zukas et al. [48], Lankarani and Nikravesh [17], Flores et al. [30], Dubowsky and Freudenberg [49], Johnson [50], Radzimovsky [51], and Goldsmith et al. [52]. It is worth noting that only the model developed by Lankarani and Nikravesh [17] is widely used in dynamic studies of mechanical multi-body systems with clearance joints. This is attributable to the simplicity of its contact force model, resulting ease of calculation, applicability to impact in multi-body systems, and rapid convergence due to the inclusion of energy dissipation modelling upon impact. In an academic monograph, Flores [53] compared different contact force models and proved the effectiveness of the model proposed by Lankarani and Nikravesh when describing the dynamic properties in the contact impact of a clearance joint. Besides, he confirmed that Lankarani and Nikravesh's model can be transformed to deal with contact problems between cylindrical surfaces.

Therefore, the continuous contact force model of Lankarani and Nikravesh was used to depict the contact impact between clearance joints in the present study. This model is expressed as

\[ F_N = K\delta^{1.5} \left( 1 + \frac{3(1 - \nu^2)}{4\delta^{-1}} \right) \]

and

\[ h_k = \frac{1 - \nu^2}{2E_k} \]

where \( \delta \) is the relative normal penetration velocity, \( \nu \) is the initial normal impact velocity, \( \nu \) is the restitution coefficient, and \( K \) is a constant dependent on the material properties of the components and their geometry. The constant \( K \) is expressed as

\[ K = \frac{4\pi(h_1 + h_2)}{(R_i + R_j)^{1/2}} \]

and velocities at the contact point have been determined, the normal contact and friction forces can be computed.

An appropriate contact force model is crucial for the precise description of the collision dynamics between bodies. So far, a variety of contact force models have been proposed by: Hertz [47], Zukas et al. [48], Lankarani and Nikravesh [17], Flores et al. [30], Dubowsky and Freudenberg [49], Johnson [50], Radzimovsky [51], and Goldsmith et al. [52]. It is worth noting that only the model developed by Lankarani and Nikravesh [17] is widely used in dynamic studies of mechanical multi-body systems with clearance joints. This is attributable to the simplicity of its contact force model, resulting ease of calculation, applicability to impact in multi-body systems, and rapid convergence due to the inclusion of energy dissipation modelling upon impact. In an academic monograph, Flores [53] compared different contact force models and proved the effectiveness of the model proposed by Lankarani and Nikravesh when describing the dynamic properties in the contact impact of a clearance joint. Besides, he confirmed that Lankarani and Nikravesh's model can be transformed to deal with contact problems between cylindrical surfaces.

Therefore, the continuous contact force model of Lankarani and Nikravesh was used to depict the contact impact between clearance joints in the present study. This model is expressed as

\[ F_N = K\delta^{1.5} \left( 1 + \frac{3(1 - \nu^2)}{4\delta^{-1}} \right) \]

and

\[ h_k = \frac{1 - \nu^2}{2E_k} \]

where \( \delta \) is the relative normal penetration velocity, \( \nu \) is the initial normal impact velocity, \( \nu \) is the restitution coefficient, and \( K \) is a constant dependent on the material properties of the components and their geometry. The constant \( K \) is expressed as

\[ K = \frac{4\pi(h_1 + h_2)}{(R_i + R_j)^{1/2}} \]

and velocities at the contact point have been determined, the normal contact and friction forces can be computed.

An appropriate contact force model is crucial for the precise description of the collision dynamics between bodies. So far, a variety of contact force models have been proposed by: Hertz [47], Zukas et al. [48], Lankarani and Nikravesh [17], Flores et al. [30], Dubowsky and Freudenberg [49], Johnson [50], Radzimovsky [51], and Goldsmith et al. [52]. It is worth noting that only the model developed by Lankarani and Nikravesh [17] is widely used in dynamic studies of mechanical multi-body systems with clearance joints. This is attributable to the simplicity of its contact force model, resulting ease of calculation, applicability to impact in multi-body systems, and rapid convergence due to the inclusion of energy dissipation modelling upon impact. In an academic monograph, Flores [53] compared different contact force models and proved the effectiveness of the model proposed by Lankarani and Nikravesh when describing the dynamic properties in the contact impact of a clearance joint. Besides, he confirmed that Lankarani and Nikravesh's model can be transformed to deal with contact problems between cylindrical surfaces.

Therefore, the continuous contact force model of Lankarani and Nikravesh was used to depict the contact impact between clearance joints in the present study. This model is expressed as

\[ F_N = K\delta^{1.5} \left( 1 + \frac{3(1 - \nu^2)}{4\delta^{-1}} \right) \]

and

\[ h_k = \frac{1 - \nu^2}{2E_k} \]

where \( \delta \) is the relative normal penetration velocity, \( \nu \) is the initial normal impact velocity, \( \nu \) is the restitution coefficient, and \( K \) is a constant dependent on the material properties of the components and their geometry. The constant \( K \) is expressed as

\[ K = \frac{4\pi(h_1 + h_2)}{(R_i + R_j)^{1/2}} \]

and velocities at the contact point have been determined, the normal contact and friction forces can be computed.

An appropriate contact force model is crucial for the precise description of the collision dynamics between bodies. So far, a variety of contact force models have been proposed by: Hertz [47], Zukas et al. [48], Lankarani and Nikravesh [17], Flores et al. [30], Dubowsky and Freudenberg [49], Johnson [50], Radzimovsky [51], and Goldsmith et al. [52]. It is worth noting that only the model developed by Lankarani and Nikravesh [17] is widely used in dynamic studies of mechanical multi-body systems with clearance joints. This is attributable to the simplicity of its contact force model, resulting ease of calculation, applicability to impact in multi-body systems, and rapid convergence due to the inclusion of energy dissipation modelling upon impact. In an academic monograph, Flores [53] compared different contact force models and proved the effectiveness of the model proposed by Lankarani and Nikravesh when describing the dynamic properties in the contact impact of a clearance joint. Besides, he confirmed that Lankarani and Nikravesh's model can be transformed to deal with contact problems between cylindrical surfaces.

Therefore, the continuous contact force model of Lankarani and Nikravesh was used to depict the contact impact between clearance joints in the present study. This model is expressed as

\[ F_N = K\delta^{1.5} \left( 1 + \frac{3(1 - \nu^2)}{4\delta^{-1}} \right) \]

and

\[ h_k = \frac{1 - \nu^2}{2E_k} \]

where \( \delta \) is the relative normal penetration velocity, \( \nu \) is the initial normal impact velocity, \( \nu \) is the restitution coefficient, and \( K \) is a constant dependent on the material properties of the components and their geometry. The constant \( K \) is expressed as

\[ K = \frac{4\pi(h_1 + h_2)}{(R_i + R_j)^{1/2}} \]

and velocities at the contact point have been determined, the normal contact and friction forces can be computed.
formula is as follows:

\[ F_T = -\frac{c_N}{C_0} \frac{v_T}{v_T} \tag{15} \]

where, \( c_N \) is a constant independent of the normal contact force, and \( v_T \) is the relative tangential velocity. In spite of its simplicity, the model ignores the case where the relative tangential velocity declines to zero. Fig. 5(a) illustrates the variation of friction with the model ignores the case where the relative tangential velocity tends to zero.

Rooney and Deravi [55] modified the aforementioned friction model to

\[ F_T = -\mu F_N \frac{v_T}{v_T} \tag{16} \]

where, \( \mu \) is the dynamic friction coefficient and \( F_N \) is the normal contact force. This model assumes that, if the relative tangential velocity at the contact point is large, the friction is calculated using the formula above; as the relative tangential velocity tends to zero, the friction lies within the following range:

\[-\mu F_N < F_T < \mu F_N\]

Threlfall [56] proposed an alternative friction force model:

\[ F_T = \mu F_N \frac{v_T}{v_T} \left( 1 - e^{-\frac{v_T}{v_T}} \right) \tag{17} \]

where, \( v_T \) is a small characteristic velocity as compared to the maximum relative tangential velocity encountered during the simulation. Compared with the friction force model presented by Rooney and Deravi, Threlfall friction force model uses a smooth curve to realize the transition between \(-\mu F_N \) and \( \mu F_N \), as shown in Fig. 5(b) and (c).

Ambrósio [57] developed a new friction force model based on Coulomb friction:

\[ F_T = -\mu c_d F_N \frac{v_T}{v_T} \tag{18} \]

where, \( c_d \) is a dynamic correction coefficient which depends on the given tolerances \( v_0 \) and \( v_1 \) for the velocity. The great merit of this friction force model, as Flores [53] said, is that the dynamic correction factor can prevent the friction force from changing direction for almost zero values of the tangential velocity, which is perceived by the integration algorithm as a dynamic response with high-frequency contents, thereby forcing a reduction in the time-step size. Furthermore, it allows the numerical stabilization of the integration algorithm. A better understanding of Ambrósio’s friction force model can refer to Fig. 5(d).

In general, the friction force models of Ambrósio, and Rooney and Deravi, are widely used to analyze the dynamic properties of multi-body systems with clearance joints. In the present study, the latter was used to model the friction between clearance joints. It is worth noting that existing friction force models are not limited to those mentioned here: additional models may be found in the bibliographical material provided.

3. Numerical example of a 2-DOF pick-and-place parallel manipulator with two clearance revolute joints

Fig. 6 shows a typical 2-DOF pick-and-place parallel manipulator which consists of a base (link 1), two driving arms (links 2 and 3) and two driven arms (links 4 and 5). In the model, links 2 and 4 are connected by a clearance revolute joint which is denoted by \( L_{-cj} \), while another clearance revolute joint \( R_{-cj} \) is used to connect links 3 and 5. The rotations of the driving arms can be individually manipulated by two servomotors, providing the end-effector with a 2-DOF translational moving capability.

Generally speaking, clearance is possibly present in each joint in a mechanism. Even if it is designed to be zero, joint clearance is also inevitable after long-term wear. Therefore, any study of the dynamic properties of such mechanical systems should include the influences of all joint clearances. However, existing studies proved that it is increasingly difficult to obtain numerical solutions of the equations governing the dynamics of these mechanical multi-body systems with multiple clearance joints. Furthermore, when encountering complex mechanical systems, numerical solutions struggle to converge. Therefore, a majority of existing studies merely consider the influences of two or three joint clearances using simple planar slider-crank mechanisms or four-bar mechanisms. In this research, the dynamic modeling of the 2-DOF high-speed pick-and-place parallel manipulator bearing clearances merely takes account of the influences of two joint clearances to reduce the difficulty in the numerical solution of model. As shown

![Fig. 5. The friction force models: (a) Dubowsky friction model, (b) Rooney and Deravi friction model, (c) Threlfall friction model, (d) Ambrósio friction model.](image)

![Fig. 6. A 2-DOF pick-and-place parallel manipulator.](image)
in Fig. 6, the two clearances selected, labeled \( L_{cj} \) and \( R_{cj} \), are located on the branch chains on right and left hand sides respectively and are mainly used to connect the master arm and slave arm. The reason why two clearances were selected lies in the fact that the inertial load variations of the two joints are more complex in the swing process of the mechanical arm. Moreover, with regard to the influence of clearance, the microscopic dynamic properties of the two joints affected the macroscopic dynamic properties of the whole manipulator. In addition, the clearances of the joints in parallel manipulators that were closer to the static platform contributed more to the kinematic error in the terminal dynamic platform and its output.

The dimensions and mass properties for the mechanism are shown in Table 1. Table 2 presents the parameters used in the dynamic simulations. Generally, it is extremely difficult to accurately obtain the friction coefficient and elastic restitution coefficient between clearance joint elements. In existing research [15,16,25,32,33,44], the two coefficients are generally estimated. In the contact force model, a value of 0.01 and 0.9 are used for the friction coefficient and coefficient of restitution, respectively.

Standards regulating joint clearance are currently unavailable: theoretically, joint clearance should be minimized to maintain the assembly ability of the joint. Moreover, in practical engineering, assembly clearance can be determined in the design process of the robot according to the requirements of kinematic accuracy and manufacturing cost. However, joint clearance is prone to increase due to wear in service. In addition, the actual joint clearances will change with fluctuations in the wear. Therefore, it is difficult to obtain an accurate actual size for each joint clearance in any mechanical multi-body system. Considering the situation above, joint clearance size is mainly estimated in existing studies. For example, Erkaya hypothesized that the clearance in the terminal joint of an industrial welding robot was 0.2 mm or 0.5 mm, but did not justify this. In the present study, the joint clearance was assumed to be either 0.5 mm or 1 mm, which may be higher than the actual value. However, the results obtained by numerical simulation successfully reflect the actual dynamic properties of a robot mechanism with joint clearance. In addition, the dynamic model for the 2-DOF high-speed parallel manipulator was established considering the influences of multiple joint clearances. As a main contribution of this study, this model is claimed to be universally applicable to the analysis of the dynamic properties of robot systems under arbitrary joint clearances. Therefore, it is suggested that the joint clearance in actual mechanical systems can be measured in advance and then substituted into the theoretical model to acquire a closer theoretical analysis result, better matching actual engineering situations.

In the numerical simulation, the time step is \( 0.00001 \) s. This value is constant and small enough for obtaining the convergence values. If the operating speed of the manipulator increases, the corresponding time step would be further reduced for achieving numerical convergence. Furthermore, the integrator scheme utilized in numerical simulation is Gear method. This method is capable of dealing with system rigidity and realizing highly efficient changeable step integration in each \( 0.00001 \) s time interval.

In the simulation, it is assumed that the manipulator is commanded in such a way as to ensure that the end-effector picks up an object from Point 1 and places it at Point 6, following the trajectory shown in Fig. 6. The motion of the end-effector along the X- or Y-axes in the global coordinate system complies with the cycloidal displacement rule given below:

\[
s = a_{\text{max}} \frac{T^2}{2} \left( 1 - \frac{1}{2} \sin \left( \frac{2\pi t}{T} \right) \right) \quad \left( 0 \leq t \leq 1 \right)
\]

where \( a_{\text{max}} \) represents the magnitude of the maximum acceleration of the end-effector along the specified axis and \( T \) denotes the time interval. The coordinates of the nodes on the trajectory in the given XOY coordinate system are

\[
P_1 \left( \frac{b}{2} - H - h \right) \rightarrow P_2 \left( \frac{b}{2} - H - \frac{h}{2} \right) \rightarrow P_3
\]

\[
\rightarrow P_4 \rightarrow P_5 \left( \frac{b}{2} - H - \frac{h}{2} \right) \rightarrow P_6 \left( \frac{b}{2} - H - h \right)
\]

where, \( H = 0.6m, b = 0.4m, h = 0.025m \). In calculation, the amplitude of the maximum accelerations of the end-effector along the \( X \)- and \( Y \)-axes are given by \( a_{\text{max}, x} = 120m/s^2 \) and \( a_{\text{max}, y} = 60m/s^2 \), respectively.

To make the end-effector realize the pick-and-place trajectory shown in Fig. 6, it is necessary to obtain the laws of motion governing the driving arms through an inverse kinematic analysis of the mechanism. For details of this issue, the interested reader is referred to the work developed by Huang et al. [58]. In a complete pick-and-place cycle of the mechanism, the variations of the rotation angular displacement, angular velocity, and angular acceleration of the driving arms are shown in Fig. 7.

### 4. Results and discussion

Generally, a joint with clearance can display three motion states with the movement of the mechanism: a continuous contact state, a separation state, and a collision state. In practice, the variations of these three motion states are affected by some specific parameters, such as the kinematic characteristics of the mechanism, the clearance size, the friction properties, the contact stiffness, and damping between joint elements. By numerical simulation, we find the system dynamic response of this five-bar parallel manipulator for two different clearance values, including variations of the clearance joint’s eccentric trajectory, the joint reaction force, the input torque, the acceleration, and trajectory of the end-effector.

Figs. 8 and 9 show the relative eccentric trajectory between clearance joint elements for a clearance sizes of 1 mm ( journal and bushing radii are 10 mm and 11 mm respectively ) and 0.5 mm ( journal and bushing radii are 10 mm and 10.5 mm respectively). Fig. 8 shows that the elements of clearance joint \( L_{cji} \) begin to separate at \( t = 0.4210 \) s, and then impact and make contact at \( t = 0.4295 \) s. With continued movement of the mechanism, the elements of this clearance joint show a second separation at \( t = 0.7564 \) s, with contact re-established at \( t = 0.7665 \) s. These two

<table>
<thead>
<tr>
<th>Bodies</th>
<th>Length (m)</th>
<th>Mass (kg)</th>
<th>Moment of inertia (kg·m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1</td>
<td>0.2</td>
<td>1.7</td>
<td>0.0084</td>
</tr>
<tr>
<td>Link 2</td>
<td>0.3</td>
<td>2.5</td>
<td>0.0233</td>
</tr>
<tr>
<td>Link 3</td>
<td>0.2</td>
<td>2.5</td>
<td>0.0233</td>
</tr>
<tr>
<td>Link 4</td>
<td>0.65</td>
<td>0.8</td>
<td>0.0282</td>
</tr>
<tr>
<td>Link 5</td>
<td>0.65</td>
<td>0.8</td>
<td>0.0282</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of restitution</td>
<td>0.9</td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>0.01</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>207 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Joint clearance size</td>
<td>1 mm, 0.5 mm</td>
</tr>
<tr>
<td>Integration step size</td>
<td>0.000001 s</td>
</tr>
<tr>
<td>Integrator scheme</td>
<td>Gear method</td>
</tr>
</tbody>
</table>
state changing processes in the clearance joint $L_{cj}$ are denoted A and B respectively. In the meantime, the elements of clearance joint $R_{cj}$ begin to separate at $t=0.5613\,\text{s}$, make contact at $t=0.5698\,\text{s}$. Afterwards, the elements of this clearance joint present the second separation at $t=0.6172\,\text{s}$ and the second contact-impact occurs at $t=0.6254\,\text{s}$. These two state changing processes in clearance joint $R_{cj}$ are denoted C and D respectively. It can also be found from Fig. 9 that, the clearance joints $L_{cj}$ and $R_{cj}$ experience two separation leaps and collisions respectively, and that the time of occurrence of the phenomenon is similar to that in Fig. 8. These results indicate that the clearance joints $L_{cj}$ and $R_{cj}$ of this parallel manipulator present two separation leaps in a complete pick-and-place working cycle, following a collision.

Collision can cause an instant increase of reaction force in the revolute joint. Figs. 10 and 11 show the variations of the joint reaction force at two different clearance sizes. The amplitudes of the reaction forces at the moments of separation and collision of clearance joint elements, namely, in the collision process of A, B, C, and D, are marked in the two figures. The joint reaction force amplitude caused by impact shows greater
variation. By comparison, the bigger the clearance value, the higher the joint reaction force amplitude caused by collision. Figs. 12 and 13 show the variations of the input torque of the driving arms over a whole pick-and-place motion cycle of the parallel manipulator. It can be found that the collision of clearance joint elements leads to an instant increase in driving torque. The bigger the clearance, the higher the peak value of driving torque caused by collision.

Figs. 14 and 15 display the translational acceleration of the end-effector of this mechanism in the pick-and-place process. It can be observed that, the bigger the joint clearance, the larger the vibration of the end-effector in the pick-and-place working process. The joint...
clearance will affect the motion stability of the high-speed parallel manipulator. Figs. 16 and 17 show the variation of the pick-and-place trajectory of the end-effector. Compared with the ideal trajectory, the actual pick-and-place trajectory of this mechanism deviates from its ideal path under the condition of containing clearances. The smaller the clearance, the closer the actual pick-and-place trajectory to its ideal version. Therefore, the joint clearances exert severe effects on the dynamic accuracy of the mechanisms.
Analysis of the simulation results revealed that two significant “separation-leap-impact” phenomena arose among clearance joint elements in a complete pick-and-place cycle. The system vibration caused by the impact reduced the dynamic stability of the system and the kinematic accuracy of the manipulator end-effector. The larger the joint clearance, the larger the system vibration caused by joint impact, and the higher the system kinematic error. Therefore, it was important to improve the dynamic properties
and kinematic accuracy of the manipulator by maintaining a close contact between clearance joint elements. Additionally, by reducing or eliminating joint clearance, the system vibration can also be effectively reduced and the dynamic properties of the system enhanced thereby. However, joint clearance is inevitable due to the presence of assembly error, manufacturing error, wear, etc. Hence, two methods were recommended in this study to decrease or suppress the influence of joint clearance on the dynamic properties of a high-speed manipulator, as indicated below:

(1) Connecting the master arm and slave arm of the manipulator using a pre-loaded spring. The tension in the spring allows constant close contact between joint elements and thus avoids the “separation-leap-impact” problem. This is an effective method of suppressing system vibration.

(2) Optimizing the pick-and-place path using a suitable pick-and-place kinematic law to smooth the variation of inertial load on the clearance joint and prevent “separation-leap-impact” events.

In the last part of this paper, the computational efficiency when simulated the manipulator with joint clearances is discussed. Usually, solution of the equations governing the dynamics of such multi-body systems with contact impact problems is slow. In the present study, the solution efficiencies of the models with their different clearances are compared. A DELL computer (CPU 31.3 GHz) is used to perform the calculation procedure. As shown in Fig. 7, the manipulator needs 0.3918 s to complete a pick-and-place action. When the joint clearance is set separately as either 1 mm or 0.5 mm, the computer used here took 517 s and 508 s to produce the solution respectively. Thus, it can be inferred that clearance size exerted little influence on the solution efficiency of the model.

5. Conclusions

The planar 2-DOF pick-and-place parallel manipulator has been widely applied in industrial automation production engineering. With the continuous development of the mechanism toward high speed and high precision, the effects of joint clearance on the dynamic performance of the mechanism cannot be neglected. The effects of joint clearance on the dynamic performance of a typical planar 2-DOF pick-and-place parallel manipulator are investigated and discussed. For modeling the mechanism, multi-body system dynamics are used. In the mathematical model, the contact effect in revolute joints with clearance is established by using a continuous analysis approach combined with a contact force model considering hysteretic damping. The contact force is calculated by Hertzian contact theory that accounts for the geometrical and material properties of the contacting bodies. Furthermore, the friction effect in clearance joints is incorporated by using a modified Coulomb friction model.

The simulation results indicate that the clearance joints present two separation leaps over a whole pick-and-place working cycle of the parallel manipulator, following a collision. The impact induces system vibration and thus reduces the dynamic stability of the system. The joint clearances affect the amplitudes of the joint reaction forces, the input torque, and the end-effector’s accelerations. The bigger the joint clearance, the higher the amplitudes of the joint reaction force, input torque and end-effector accelerations caused by collision will be. Furthermore, the joint clearances degrade the kinematic and dynamic accuracy of the system, and affect the motion stability of the high-speed parallel manipulator.

It proposed two approaches to reduce the influence of joint clearance on the dynamic properties and dynamic accuracy of the 2-DOF high-speed pick-and-place parallel manipulator, as indicated by the following: connecting the master arm and slave arm of the manipulator using a pre-loaded spring. The tension in the spring allowed constant close contact between joint elements and thus avoided the phenomenon of “separation-leap-impact” which proved effective in suppressing system vibration. Optimising the pick-and-place force using a suitable kinematic law to improve the variation of the inertial load on the clearance joint and prevent “separation-leap-impact” events. Both methods offer significant improvements in the high-speed pick-and-place performance of the manipulator and reduce its vibration amplitude.

Acknowledgements

The authors would like to express the sincere thanks to the referees for their valuable suggestions. This project is supported by National Natural Science Foundation of China (Grant No. 51305300), Natural Science Foundation of Tianjin (Grant No. 13JCQNJC04500), Tianjin Higher Educational Science and Technology Foundation Planning Project (Grant No. 20120407) and the Scientific Research Starting Foundation of Tianjin University of Technology and Education (Grant No. KYQD11008). These supports are gratefully acknowledged.

Appendix

The generalized coordinate vector of the planar 2-DOF pick-and-place parallel manipulator is given by

\[
\mathbf{q} = \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 + \frac{1}{2} \cos \theta_2 + \frac{1}{2} \\ y_2 + \frac{1}{2} \sin \theta_2 \\ x_3 - \frac{1}{2} \cos \theta_3 - \frac{1}{2} \\ y_3 - \frac{1}{2} \sin \theta_3 \\ x_4 + \frac{1}{2} \cos \theta_4 - x_5 + \frac{1}{2} \cos \theta_5 \\ y_4 + \frac{1}{2} \sin \theta_4 - y_5 + \frac{1}{2} \sin \theta_5 \\ \theta_2 - \theta_2^0 - \omega_2 t \\ \theta_3 - \theta_3^0 - \omega_2 t \end{bmatrix}^T
\]

(1)

The mass matrix of the system is given by

\[
\mathbf{M} = \text{diag}[m_1, m_1, l_1, m_2, m_2, l_2, m_3, l_3, m_4, l_4, m_5, l_5]
\]

(2)

The restriction equation of the system is

\[
\Phi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0
\]

(3)

where, \(l_i (i = 1, 2, 3, 4, \text{ and } 5)\) refers to rod length, \(\theta_i^0\) and \(\theta_i^0\) denote the initial angles of the master arm, and \(\omega_2\) and \(\omega_2\) are the driving angular velocities of the master arm.

The generalized external force vector of the system is

\[
\mathbf{Q}^A = \begin{bmatrix} 0 & -m_1 g & 0 & f_2^* - m_2 g + f_2^* & T_2 & f_3^* - m_3 g + f_3^* & T_3 \\ f_2^* - m_2 g + f_2^* & T_2 & f_3^* - m_3 g + f_3^* & T_3 \end{bmatrix}^T
\]

(4)

where, \(f_i (i = 2, 3, 4, \text{ and } 5)\) refers to the contact reaction force of the clearance joint, \(T_i (i = 2, 3, 4, \text{ and } 5)\) represents the moment of \(f_i\) about each rod, and gis the gravitational acceleration. By substituting Eqs. (2)–(4) into Eq. (5), the multi-body dynamic
equation for the 2-DOF pick-and-place parallel manipulator is obtained.

\[
\begin{bmatrix}
\mathbf{M} & \mathbf{F}_q^T \\
\mathbf{0} & \lambda
\end{bmatrix}
\begin{bmatrix}
\mathbf{q} \\
\dot{\mathbf{q}}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{Q}_c \\
\mathbf{0}
\end{bmatrix}
\]

(5)

where, \( \dot{\mathbf{q}} \) is the acceleration vector of the constraints, \( \lambda \) is a vector of Lagrange Multipliers, \( \gamma = - (\mathbf{F}_q(\mathbf{q}) - \mathbf{2}\mathbf{F}_q \cdot \dot{\mathbf{q}} - \dot{\mathbf{Q}_c}) \) is vector that groups all the terms of the acceleration constraint equations that depend on the velocities.

References


